

# Generative Design Systems for the Industrial Design of Functional Mass Producible Natural-Mathematical Forms

Axel Nordin\*, Andreas Hopf\*\*, Damien Motte\*\*\*

\* *Division of Machine Design, Department of Design Sciences, Lund University, Sweden,  
axel.nordin@mkon.lth.se*

\*\* *Division of Industrial Design, Department of Design Sciences, Lund University, Sweden,  
andreas.hopf@design.lth.se*

\*\*\* *Division of Machine Design, Department of Design Sciences, Lund University, Sweden,  
damien.motte@mkon.lth.se*

Abstract: Nature provides us with a vast source of inspiration. However, given industrial designers' open-mindedness and inquisitiveness, a surprisingly limited set of nature-derived symbols continues to be popular in this creative discipline. Rather than designing products mimicking nature, it is probably more rewarding designing them based on the natural principles leading to its growth and form. However, the constraints related to mass produced products make designing with the often complex forms found in nature a daunting task for a human designer. In this paper, we demonstrate, through the implementation of two generative design systems, how fairly complex everyday objects based on three-dimensional natural-mathematical morphologies can be designed, evaluated and produced using mass production techniques; that digital and analogue methods can be linked to create an aesthetic and functional whole beyond purely decorative mimicry. The output from the generative design system made it possible to produce a fully developed, "ready-for-sale" product, with potential for large-scale production. This is a step towards enabling industrial designers the same level of form articulation as has been available to artists and architects, even though the constraints on the design activity are much different.

**Key words:** *Generative design system, L-system, minimal surface, genetic algorithm*

## 1. Introduction

Nature is, and always has been a source of inspiration for human artistic endeavors. This is not without its reasons as all organisms are both image and result of evolution, their forms being the diagrams of invisible natural forces, as Thompson [28] concluded. However, given industrial designers' open-mindedness and inquisitiveness, a surprisingly limited set of deficitary symbols (leaf, tree, hexagon, the colors green and blue, etc.) continues to be popular in this creative discipline. In many cases, unfortunately, nature-inspired design serves as specious greenwashing ingredient in marketing strategies, bestowing a sustainable aura on sometimes very unsustainable products and services. Rather than designing a roof mimicking a snowflake, it is probably more rewarding designing it based on the natural principles leading to its growth and form.

Within design research, the use of morphologies (i.e. forms, shapes or structures) derived from nature has been the object of study for quite some time (see e.g. [22]), however, it has not received the same amount of attention as in for instance art or architecture. The reasons might partly be found in differences in attitudes between the fields, but more importantly in the technical constraints unique to product design. Whereas architectural or artistic

projects implementing complex morphologies are most often built in one exemplar with already defined customer and financial resources, the reality for a product based on similar principles is vastly different. Unless the objective of the industrial designer is to create a one-off art-design to be sold to one or very few customers, the design needs to conform to mass production systems, and reliably perform its function. These constraints make the use of complex morphologies, such as the Voronoi-diagram, minimal surface or Lindenmayer systems in design a daunting task. Already time-consuming manual considerations of for instance producibility and structural stability of the product become insurmountable if the form is complex. Although computational methods exist for analyzing arbitrary geometry, they require a highly specialized set of skills which the industrial designer rarely possesses. Instead, industrial design teams in larger companies rely on the expertise of design engineers to evaluate their designs. While this specialization is most often non-problematic, a design project involving complex forms might require many iterations between the departments, leading to increased development time and cost. Having many iterations might also hinder creativity, as trying radically new ideas is associated with high costs.

[19] and [20] suggest using an *interactive generative design system* to aid the designer in the use of complex morphologies. The handling of the form is integrated with computational evaluation systems and constraint handling systems which automatically can steer the form creation towards a solution which is functional, structurally stable, and producible. This approach has been applied to 2.5D objects such as bookshelves and tables using 2D tessellations such as the Voronoi-diagram and isohedral tessellations. The results show that this approach is feasible; *however, it is uncertain whether the approach is applicable to fairly complex three-dimensional forms, to other product typologies and to other manufacturing technologies as well.*

In this paper, we therefore examine the extent of the applicability of the approach by investigating algorithms originating in nature that allow for adapting complex three-dimensional forms to functional, manufacturing, and aesthetic constraints and objectives. It was also important to demonstrate that the output of nature-based computational means of form generation does not need to be confined to rapid prototyping, but can also be *realized with established fabrication technologies allowing for mass production* - whether using high or low tech materials. The forms are complex in the sense that creating them manually would be very time consuming and difficult; the constraints and objectives to which the forms must adhere further adding to the complexity of generating feasible forms.

In this paper, we apply the approach from [19] and [20] for two three-dimensional forms. We examined 1) Lindenmayer systems (L-systems) coupled to a genetic algorithm (GA) to create user-controlled branching support structures, and 2) minimal surfaces to create user-controlled lighting diffusers. We chose these morphologies based on their functional and aesthetic properties, and to represent two vastly different adaptation processes. Both these applications have a set of constraints and objectives associated with them in terms of functionality and manufacturability, described in Section 3. To verify this process a set of lights based on the minimal surfaces have been built and have been selected for exhibition at several international design fairs (DMY 2011, Stockholm Furniture Fair 2013, Biennale Internationale Design Saint-Etienne 2013) showing that the generative design system made it possible to produce a fully developed, "ready-for-sale" product, with potential for large-scale production. These studies reinforce the feasibility of using both forms derived from nature and their generation methods for product design.

## 2. Related works

Complex natural-mathematic morphologies have been the subject of interest in the artistic world for quite some time with artists such as Herbert W. Franke and Peter Henne creating algorithm-based computer graphics already in the 1950s and early 1960s. Within computer based bio-inspired generation of art and music, works by Sims [27], Todd and Latham [29] and Romero [24] show the wealth of research conducted on the topic.

In architecture, Lynn [16], Kolarevic [12] and Oxman [21] represent important works showing the extent to which digital design tools and generative design have been adopted by the architectural community.

Some works in design computing are taking into account functional or technical constraints and aesthetics. Shea and Cagan [26] use a combination of shape grammar and simulated annealing for both functional and aesthetic purposes and applied it for truss structures. Their model is re-used in [14] (shape grammar and GA) to develop stylistically consistent forms applied to the design of a camera. The designs generated took into account the constraints linked to the spatial component configuration. A designer was in charge of the aesthetic evaluation, following the interactive genetic algorithm paradigm [11]. Common are also the use of evolutionary methods to optimize a parameterized geometry in relation to objectives such as minimize weight or structural rigidity (see e.g. [1], [2]).

Examples of natural-mathematical morphologies in product design can mostly be found in industry with examples such as Trubridge's polyhedral ceiling lamp [30] and Wertel and Oberfell's Fractal-T [31] table. Many of the products based on complex morphologies are, however, produced by rapid prototyping, many examples can be found at rapid prototyping providers such as Shapeways ([www.shapeways.com/](http://www.shapeways.com/)) where consumer can also customize products such as jewelry developed by for instance Nervous System (<http://n-e-r-v-o-u-s.com/>). Products realized with traditional materials include Kram/Weisshaar's Breeding Tables project, which generates variations of a table design using a GA that modifies a set of parameters ruling the support structure [13]. The system does not take stability into account, but it does ensure the producibility of the designs through constraints on the parameters. The Computational Chair project developed by EZCT Architecture & Design research [6] also uses a GA to generate design variations of a chair built from pieces of plywood glued together, but the algorithm in this case also minimizes the weight and ensures the structural stability of the chair through finite-element analysis. These examples show that although complex morphologies have been used in design, they are most often items which have no or few constraints such as structural stability, and are made by the process of rapid prototyping which permits almost arbitrary shapes to be produced, but is so far prohibitively costly for the production of larger structures such as tables or chairs, and unless specialty techniques and materials are used lack the surface quality and structural strength required for useful objects. Examples such as Kram/Weisshaar's tables and EZCT Architecture & Design research's chairs are built with traditional materials and do take into consideration the structural rigidity of the product, but are not mass produced objects, but rather one-off art-designs.

In this work, we therefore focus on generating designs realized with established fabrication technologies allowing for mass production based both on the use of natural forms and on the use of the adaptive natural processes for taking into account manufacturing constraints, functionality and aesthetic properties.

### 3. Implementation

#### 3.1. Support structure with L- systems and GA

##### L-systems

An L-system, is a shape grammar ruling how a structure grows, used for example to model the growth of plants and some organisms [15], [23] (see Figure 1 for examples of structures in nature that are typically modeled with L-systems). L-systems have been used for artistic purposes and linked to GAs in for instance [17] and [9]. L-systems can also be used to generate self-similar fractals. L-system-based two- or three-dimensional structures have an irregular branch-like formal aesthetic and connect points on the plane or in space from a central node. An L-system is defined by a set of variables or sub-segments that can be used in the structure, a starting point, and a set of rules describing in what way the sub-segments can be combined. Their infinite variability makes them suitable for highly individualized yet self-similar objects. L-systems are useful for the generation of structures reaching to points in space or target points on a space envelope or for the generation of spatial lattices assembled from a possibly limited number of discrete elements. Because of their irregularity, objects based on L-systems are preferably manufactured via additive fabrication, laser/water jet cutting with subsequent computer numerically controlled (CNC) bending, and to a certain degree CNC milling.

Even with relatively few possible angles and lengths, an L-system can generate millions of possible branching structures, which in itself is not a problem until functional and manufacturing constraints and objectives come into the picture. Given objectives and constraints it becomes apparent that a human designer would require many years to sift through and evaluate the possible branching structures stemming from the L-system definition. What is needed is an optimization algorithm that does this automatically and quickly while leaving the user to evaluate the qualitative aspects - such as aesthetics - of the few solutions satisfying the constraints. There are numerous algorithms for optimizing structures; one of the most frequently used algorithms for non-linear modular structures is the GA [7]. A GA is a search heuristic that mimics the process of natural evolution. A GA treats each candidate solution as an individual, encoded by its genotype. Together, the individuals create a population of candidate solutions. The individuals in the population are then, depending on their fitness in relation to the constraints and objectives, modified by processes such as inheritance, mutation, selection, and crossover to create the next generation of the population.



Figure 1. L- systems in nature: Mangrove roots<sup>1</sup>, Redwood branches<sup>2</sup>, Fern leaf<sup>3</sup>

## Application

An L-system is ideal for creating structures from a limited set of elements, adhering to some rules. Therefore, an L-system was applied to create a branching structure composed of individual pieces manufactured by laser cutting and CNC-bending sheet metal (see Figure 2). In this application the goal was to create a structure that would connect a point in space to a plane, similarly to a support structure for a roof. To limit the number of types of pieces needed to be manufactured, the pieces used should be discrete, meaning their length and angles should have a limited number of possible values. In this application the constraints were: no intersection of the branches, no intersection between the branches and the support surface and a certain required number of branch ends; the objective was that the ends of the branches should be as close as possible to the support plane.

## Implementation

There are many applications available for generating L-systems, such as Branching [18], powerPlant [25] or L-studio [10]. However, few support export to common 3D file-formats, and none can be scripted to evaluate the properties of the L-system. Therefore a custom L-system generating script was implemented in Matlab to gain full control of the structure. Matlab has a wide array of optimization tools built in, which makes the connection of the L-system generation to the GA efficient. The L-system used for this application consists of a fixed starting point, a number of branches with different lengths and angles, which in turn have a number of branches connected to their ends, and so on. The L-system script takes as input the requested number of branching levels in the structure, the maximum number of new branches at the end of each branch, the branch lengths allowed, and the branching angles allowed. The script takes this input and creates random branching structures from the data by combining different branch lengths and angles.

A GA was used to find satisfactory branching structures. The GA used is the standard Matlab implementation with rank as scaling method, stochastic uniform as selection method, Gaussian as mutation function, single point as crossover function, elite count 2, and crossover fraction 0.8. The GA was run with a population of 150 individuals, during a maximum of 1500 generations. A GA represents an individual as a genotype. In this case the

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<sup>1</sup> © 2003 Cesar Paes Barreto, available at <http://www.sxc.hu/photo/40192>

<sup>2</sup> © 2011 Lukas Osinski / CC BY-SA 3.0, available at [commons.wikimedia.org/wiki/File:Metasequoia\\_glyptostrobooides\\_Marki\\_branches.jpg](https://commons.wikimedia.org/wiki/File:Metasequoia_glyptostrobooides_Marki_branches.jpg)

<sup>3</sup> © 2008 Forest and Kim Starr / CC BY-SA 3.0, available at [commons.wikimedia.org/wiki/File:Starr\\_080117-2229\\_Microsorum\\_musifolium.jpg](https://commons.wikimedia.org/wiki/File:Starr_080117-2229_Microsorum_musifolium.jpg)

genotype consisted of one instance of the L-system, i.e. a list of building instructions such as "add a branch with length 100 and angle  $30^\circ$  to branch 2". This genome is then interpreted by the GA to create a phenotype, in this case the 3D geometry which was evaluated by the system. The GA could mutate the branching structures by shifting their branch lengths and angles between the predefined values. The crossover has been done by grafting random parts of two branching structure parents into one child structure. The evaluation function scores each structure in accordance to how well it fulfills the constraints and objective. Once the optimization has ended, the resulting solutions satisfying the constraints and minimizing the objective can be visualized on screen, reviewed by the industrial designer, and sent to a surface modeling software where drawings of the parts could be created and used for fabrication (see Figure 3).



Figure 2. Structure mock-up of one of the generated L-systems

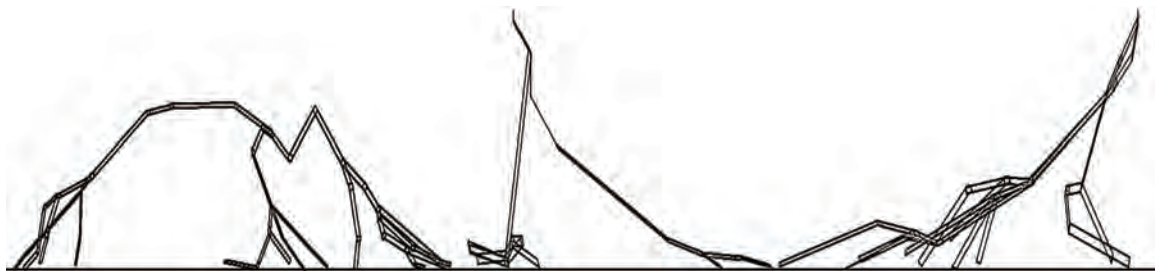


Figure 3. Output of the optimization process showing three different solutions that satisfy the constraints

### 3.2. Controlled lighting diffusers with minimal surfaces

#### Minimal surfaces

Minimal surfaces were originally a name for surfaces that minimized surface area, subject to some constraint, such as total volume enclosed or a specified boundary, with research on the subject dating back to the 18th century (see e.g. [8, pp. 1-5]), but the term is now used more generally to describe a surface with a mean curvature of zero [4]. Minimal surface based three-dimensional structures have self-similar tent-like formal aesthetic. They are

spanning connected boundaries in space. Their infinite variability makes them suitable for highly individualized yet self-similar objects. Minimal surfaces are useful for: the generation of lightweight load-bearing tensile structures, the finding of area and weight minimizing surfaces within given boundaries. Architects such as Frei Otto and Barry Patten and artists such as Robert Engman and Robert Longhurst have used these properties of minimal surfaces before computational methods became prevalent. More recently, projects such as that of Design Research Exchange[5] show how modern software for form generation and structural evaluation can be combined for high-rise structures. Because of their irregularity, objects based on minimal surfaces are preferably manufactured via additive fabrication, CNC milling, vacuum forming and to a certain degree via concrete casting, metal casting, metal stamping, slip casting.

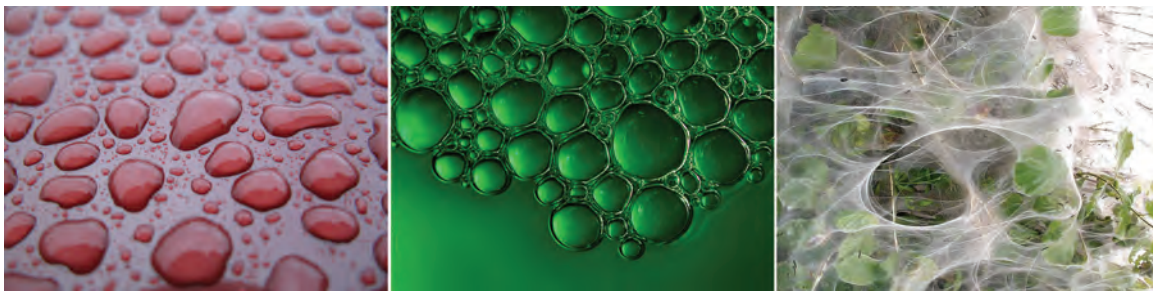


Figure 4. Minimal surfaces in nature: raindrops<sup>4</sup>, foam<sup>5</sup>, caterpillar webs<sup>6</sup>

### Application

In order to utilize the properties of the minimal surface, surface area minimization, while keeping the volume constant, slip cast porcelain shapes were deemed suitable. Because of porcelain's translucency and its matte surface when unglazed, the shapes were used for reflecting and diffusing light from high powered LEDs. The surface minimization yields surfaces which are optimal in terms of material usage for containing a certain volume. We wanted to use surface minimization to simulate a drop of water with a user-defined bottom contour resting on a flat surface. In order for a user of the application to be able to control the shape, the 2D contour of the initial shape should be possible to modify (see Figure 5), and the resulting shape of the surface minimization should be easy to review. The objective is thus to minimize the surface, while constraining the contour and volume of the shape.

### Implementation

To generate and manipulate a minimal surface there are a few alternatives, the analog way that architects such as Frei Otto and Barry Patten used consists in using materials that seek to minimize their surface energy, and thereby generating minimal surfaces by their physical properties such as elastic fabrics, liquids, and soap films that are constrained by boundary wireframes. However, this method is time-consuming and difficult to control,

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<sup>4</sup> © 2007 Andrew Bossi / CC BY-SA 2.5, available at [commons.wikimedia.org/wiki/File:2007\\_10\\_25\\_-\\_Greenbelt\\_\\_Water\\_drops\\_on\\_a\\_Saab\\_9-3\\_roof\\_2.JPG](https://commons.wikimedia.org/wiki/File:2007_10_25_-_Greenbelt__Water_drops_on_a_Saab_9-3_roof_2.JPG)

<sup>5</sup> © 2007 woodleywonderworks / CC BY-SA 3.0, available at [www.flickr.com/photos/wwwworks/667298782/](http://www.flickr.com/photos/wwwworks/667298782/)

<sup>6</sup> © 2006 Penny Mayes / CC BY-SA 2.0, available at [commons.wikimedia.org/wiki/File:Abstract\\_art\\_in\\_the\\_hedgerow\\_-\\_geograph.org.uk\\_-\\_178953.jpg](https://commons.wikimedia.org/wiki/File:Abstract_art_in_the_hedgerow_-_geograph.org.uk_-_178953.jpg)

and does not easily translate to technical manufacturing instructions. A more efficient and versatile method is to generate the surfaces digitally through software written to simulate the physics of surface tension and gravitation.

Many programs for generating minimal surfaces exists, such as Ken Bracke's Surface Evolver (SE) [3] that is versatile and powerful. However, it requires that the input to the application is scripted in a specific language. In order to be able to easily control the contour and height of the form, without the user having to hard-code geometry into a SE-script, a custom written script in Matlab was created which takes as input a contour of a 2.5D volume (see Figure 5a), and outputs a script directly to SE. In this application the code tells SE to treat the input geometry as a volume of water resting on a surface under the influence of gravitation and a wetting angle between the liquid and the surface. The geometry resulting from SE's minimization of surface energy can be displayed on-screen for evaluation by the designer (see Figure 5c). It enables the user to sketch and modify the contour and height of the shape, and then get instant feedback of the resulting minimal surface. If the surface is deemed interesting it can be exported for use with all major surface modeling software such as Rhino or Alias. Using a surface modeling software, thickness can be added to the surface (see Figure 6a) and then the thickened shell can be used as input to 3D-printing software, or computer-aided manufacturing (CAM) software such as ArtCam, which are then able to generate instructions for a CNC mill to cut the shape from a block of model material. The milled or 3D-printed model can be used as is (see Figure 6b), or used as master models for creating plaster molds for casting ceramics and plastics (see Figure 6c).

Three minimal surface master models were produced using this workflow, two were 3D-printed, and one was CNC-milled. The cost of 3D-printing and other similar rapid prototyping techniques is still somewhat prohibitive when outputting large shapes, and it is therefore the largest shape was milled. The durability of the 3D-printed models when in contact with moisture also make them less than ideal, however, it should be noted that the surface quality generated by rapid-prototyping is often good enough to use directly for mold making without any extra finishing needed, whereas the surfaces resulting from milling might require sanding and filling. The three shapes were then used as master models for making plaster molds, which in turn were used to cast ceramic shells that were later fired (see Figure 7).

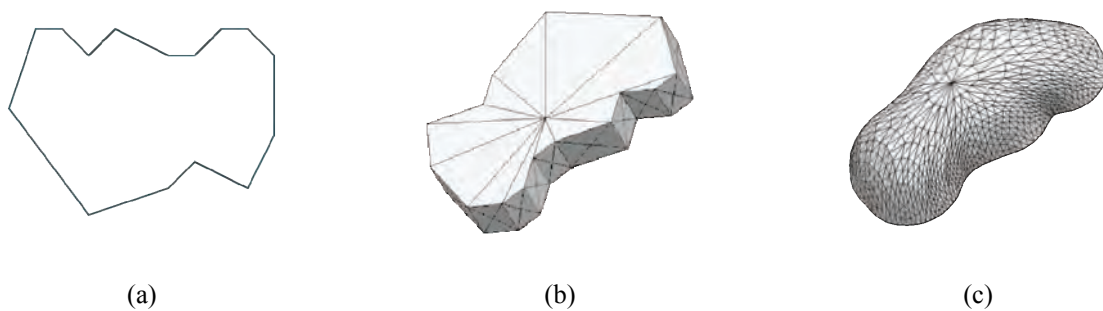


Figure 5. a) User defined drop contour, b) initial extruded surface block, c) first surface energy minimization step



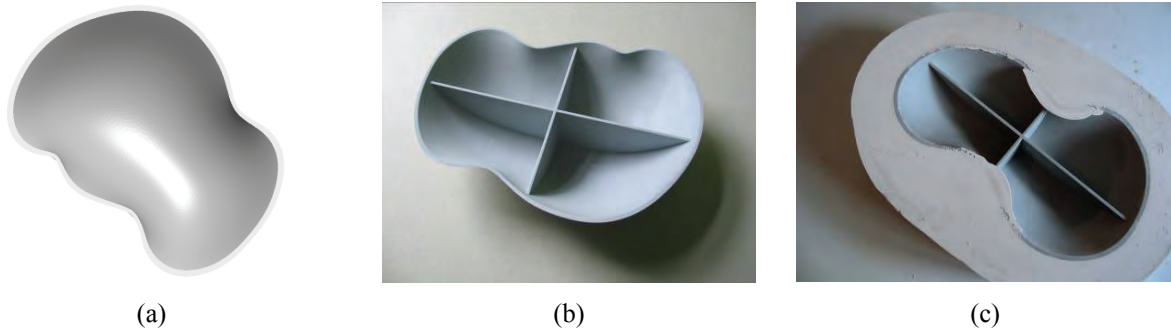


Figure 6. a) Thickened mesh, b) 3D printed model with support ribbing, c) mold-making



Figure 7. Final pieces after firing

#### 4. Discussion and conclusion

This paper has demonstrated that fairly complex everyday objects based on three-dimensional natural-mathematical morphologies can be designed, evaluated and produced into a solid form; that digital and mass production methods can be linked to create an aesthetic and functional whole beyond purely decorative mimicry. We showed that the approach is not limited to two-dimensional morphologies and 2.5D objects by describing a software-based process to design with three-dimensional morphologies which was used to generate porcelain diffusers for high-power LED lighting, and stainless steel support structures (pending realization), thus further validating the approach proposed in [19] and [20]. In the case of the porcelain diffuser, the output from the generative design system made it possible to produce a fully developed, "ready-for-sale" product, with potential for large-scale production. This is a step towards enabling industrial designers the same level of form articulation as has been available to artists and architects, even though the constraints on the design activity are much different. The user could easily change the inputs to the algorithm, either through text-based or graphical interaction, and get feedback of the resulting forms. For the minimal surface application, the feedback of the optimized shape was almost instantaneous, generally requiring less than a second for returning an optimized shape. The optimization process for the L-system required more time, usually around 3 minutes to converge on a satisfactory solution. The design tools could find solutions that satisfied the constraints. For the minimal surfaces, the intended and final volumes were identical. The structures generated by the L-system and GA all complied with the constraint that no parts should intersect. In terms of optimization the surface minimization gave material savings ranging from 31%-23% between the original form defined by the user, and the resulting SE output, while keeping the volume constant, while the L-system optimization could find structures that were on average 90% closer to the support surface than the starting L-system.

Access to an enhanced morphological repertoire, exploiting fully the possibilities to design with nature-derived forms, and, resultantly, enhanced creativity could benefit industrial designers. Other benefits can be envisioned: the emergence of digital crafts might enable relocating production to the vicinity of consumption, and rededication of existing production methods and equipment to produce individualized products could become a reality. In the extension of our approach, introducing generative design tools to consumers might lead to participatory and community-based designing, on- or offline, linked to digital fabrication.

Realizations from industrial designers often take place in the industrial context of product development. Compared to a regular product development process, it is clear that an approach such as that described in this paper can be beneficial for creating products that are individualized for every application. However, given the extra effort required to implement a generative design system for a chosen morphology and product, it might not be economically feasible for mass production. In this paper, two algorithms for form generation found in nature have been demonstrated, but the approach could be used with morphologies from other origins. Future topics of research include how applicable the approach is to more complex products such as dynamic systems.

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